

Mathematical Justifications For Common Observations in AA

By

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Introduction

In [1], the mathematics of continuously changing objects was used to compare an average alcoholic with an intellectual one, each getting AA treatment from the start. It shows how different the behavior of each is at various milestones and how important it is to look at each member individually.

In this paper, we compare an AA member progressing naturally with the same member getting AA treatment from the start. We will be able to give a mathematical justification to:

1. why so many members go out over and over again.
2. why the 90 day landmark is the place where AA treatment becomes fixed for the rest of the person's life.
3. why alcoholics who have been sober for a long time go out.

Finally we give a new interpretation of the quartiles discussed in [1] and a deeper understanding of the emotional levels involved in the steps.

The algorithms involved in the paper will be given in the appendices at the end.

Preliminaries

In the paper, "A Mathematical Journey From Despair to Happiness Through the Fellowship of A.A.", referred to as [1] in this paper, we listed ten measurable activities (macs):

M(morning), W(work), Ex(exercise), Sl(sleep), L(leisure), Re(religion), Se(sex), Ax(anxiety), Ar(artistic), C(conversation).

When they are given with definite time values whose sum adds to a fixed time period (in this paper, one day), we call that a measurable activity sequence(Mas). We find a Mas for our initial state Q(despair) and one for our ideal state \mathcal{A} (happiness). Each Mas gives rise to a four dimensional vector of positive numbers whose coordinates represent the P(physical), (Intellectual), E(emotional), S(spiritual) parts of ourselves at that time. This vector gives rise to a distribution [Q] obtained by dividing each dimension by the sum of its dimensions, representing the percent of time given to P.I.E.S respectively. Whenever a vector with positive entries is to be looked at as a distribution, we put it in brackets,.

The distribution of the ideal Mas leads us to a unique special matrix A, called an activity matrix, whose eigenvector from its largest eigenvalue has positive entries and a distribution denoted by [A] which is identical to [\mathcal{A}]. This matrix A is called the identity(ID)matrix since it represents who the person identifies himself as with respect to the dimensions. The matrix will be obtained from a given Mas using algorithm 1, given in appendix (a).

Once we have the Mas for Q and the Mas for A, we can find the Mas for A^nQ for any whole number n, allowing us to trace the recovery of the alcoholic at any time during his treatment by AA, which is really a life long process.

For an alcoholic, the dry drunk state begins the day the alcoholic stops drinking. His negative emotions Ax are extremely high, especially his obsession to drink; his rationality Ar is much less than it would normally be, and it will take quite some time before he can begin to think in a more rational way.

Measurable Activity Distributions

	P	I	E	S
M(morning)	.4	.25	.2	.15
W(work)	.6	.2	.15	.05
Ex(exercise)	.5	.2	.25	.05
Sl(sleep)	.55	.14	.065	.245
L(leisure)	.085	.315	.2	.4
Re(religion)	.1	.25	.15	.5
Se(sex)	.12	.15	.45	.28
Ax (anxiety)	.02	.25	.63	.1
Ar(artistic)	.02	.7	.18	.1
C(conversation).	.02	.6	.32	.06

Table 1

Just as scientific terms such as work, energy, force, pressure, etc, are defined by their mathematical formulae and not by their popular meaning, so it is with the measurable activities. They are defined by their distributions. By examining these distributions, we are led to an understanding to what a particular mac represents in our daily lives over and above its common usage.

We are on safer ground with the dimensions, where we can more readily identify our physical, intellectual, emotional, and spiritual parts of ourselves. They represent our basic elements whereas the macs represent compounds whose formulae are the distribution of the elements.

We must first adjust our common notion with our mathematical definitions. M is a misnomer, It stands for morning, ie.getting ready to meet the day. It should have been, getting ready to meet the event. It is the day, but it could also include getting ready to go out to dinner, or the theater, or anywhere you go where you change your clothes, shower, etc. Likewise Sl doesn't just involve our primary sleep time, but includes taking naps. W is necessary whereas Ex is chosen. Both are highly physical but their motives differ sharply. Leisure time is probably the most misunderstood. It does not mean relaxing and doing nothing, because it has a high spiritual percent .4. Therefore it is not far from Re which has a spiritual percent of .5 but less intellect and less emotions.

Any action that we take can be regarded as a mixture of macs in keeping with the analogy to chemistry.

Suppose for example you are driving to work in during rush hour traffic. Such an act involves W (work) in the sense of keeping your eyes on the road and on the car in front of you as well as operating the vehicle. You certainly have some heightened Ax (anxiety), not only fear of being hurt or killed by some errant driver, but you may be late for work. Your frustration mounts as traffic comes to a halt and you have no idea why this is happening. Ar comes in when you attempt to clear up what is keeping everyone from moving. Your eye strains to see way ahead of you what the cause might be.

You turn on the radio to get away from the reality you are living to find solace in what is happening elsewhere in the world or you listen to music to soothe your anxiety. In either case you are introducing L (leisure) into the action. You might also say a prayer for the highway mess to clear up quickly. Thus we have a mixture of macs with given time periods identifying this experience for you. In order to use such a mixture mathematically, it is necessary to assume the following axiom:

Axiom3. At any given instant, the mind is occupied with exactly one mac. The next example should make this clear.

You are at the Fitness Center exercising on the bicycle. You pedal for one half hour straight without ever stopping. Have you exercised for that time? The answer is no. While you were pedaling you checked your heart rate, bring in Ar . It was higher than it normally is bringing in Ax , and you slowed your pace a bit. You then noticed the pretty blond in the aisle in front of you with tight sweat pants. You get somewhat aroused bring in Se . You turn on the TV bringing in L , and the guy on the bicycle next to you starts chatting bringing in C .

Thus we have a mixture of these macs and what the axiom is saying is that only one of them occupies your mind at any instant. Two or more may move quickly in and out giving you the feeling that they are simultaneous, but they are not. At the end of the half hour each mac in the mixture will have been in your mind a definite amount of time, so when we add it all up it comes to one half hour. Of course your body will keep reminding you that you are exercising and so the lion's share of time will go to Ex , but certainly the other macs will take their due.

The Matrices

The activity matrices that we will use will stem from the same model as used in [1].

$$M = \begin{bmatrix} 1 & .2726 & .4641 & .6177 \\ .142 & 1 & .3004 & .3998 \\ .1182 & .1469 & 1 & .3327 \\ .1122 & .1394 & .2373 & 1 \end{bmatrix}$$

The model M was derived so that the normal person N, the ID matrix of the average person, would have equal main diagonal elements and a Mas that would match as closely as possible what we expect of the average person's daily life to be in our society, which is

M.	W.	Ex.	Sl.	L.	Re.	Se.	Ax.	Ar.	C
1	6.7	.6	7	2.9	.48	.42	2.1	1.15	1.65

The numbers are given in hours. Thus $[N] = [.3755 \ .2547 \ .1958 \ .1740]$

All matrices will be of the form:

$$A(u_1 u_2 u_3 u_4) = \begin{bmatrix} 1-u_1 & .2627u_1 & .4641u_1 & .6177u_1 \\ .142u_2 & 1-u_2 & .3004u_2 & .3998u_2 \\ .1182u_3 & .1469u_3 & 1-u_3 & .3327u_3 \\ .1122u_4 & .1394u_4 & .2723u_4 & 1-u_4 \end{bmatrix}$$

The set of all such matrices will be denoted $\text{Pop}(u)$ (population with respect to u), where $u = (u_1 u_2 u_3 u_4)^{1/4}$, the geometric mean of the u_i 's, $0 < u_i \leq 1$.

As shown in [1], the population of AA members has $u = .01$, where an application of each matrix in $\text{Pop}(u)$ takes 18 days on a 360 day year which means that $n/20$ specifies the number of years, giving us a time measurement to examine when things happen.

Thus, we have

$$N(.01) = \begin{bmatrix} .99 & .002726 & .004641 & .006177 \\ .00142 & .99 & .003004 & .003998 \\ .001182 & .001469 & .99 & .003327 \\ .001122 & .001394 & .002373 & .99 \end{bmatrix}$$

for our normal alcoholic, and

$$G(.02 \ .003 \ .01 \ .015) = \begin{bmatrix} .98 & .005452 & .009282 & .012354 \\ .0004733 & .996 & .001001 & .001333 \\ .001182 & .01469 & .99 & .003327 \\ .001683 & .002091 & .0035595 & .985 \end{bmatrix}$$

for our intellectual alcoholic.

The Alcoholic's natural Progression

According to Axiom2.in [1],once a person stops drinking (a dry drunk), he will move in the direction toward his ideal distribution whether he joins AA or not.The difference will be in the time it takes him to reach various milestones.If he does join AA and feels good about being among people who care about him and really wants to stop drinking, we can let his ID matrix have $u=.01$ and use the 18 day time for each application. However AA has a long list of suggestions for new members.

AA suggestions

Changes In Mas

Get a sponsor, get a home group, and get active.
Go to 90 meetings in 90 days,

More Ar,W
More W,Re.
Much less L

Keep your mouth shut because you don't know what
You are talking about.

Much less C

Read the Big Book. Read the book on steps and traditions.

More Ar

Listen to speakers.

More Ar, Re

Work the steps.

More Ar,W,Re

Whatever one's natural progression is at the 90 day mark, AA is requesting more W, much less L, more Re, much more Ar, and much less C. Nevertheless, the two must be equivalent at the 90 day mark, ie.have distributions that are within 1minute in each coordinate to what the matrix shows.

However he is also told from the start that there are no demands in AA. Everything is a suggestion, not a requirement. It is quite natural for him to think of AA as an international club where he can attend any meeting he wants any time he wants, be warmly welcomed and not even be required to pay any money to attend. It is a place he can go and relax and hear interesting stories, once he is able to overcome the stigma of AA as a large group of drunks of which he is a member. He can also tell himself that the treatment is really simple, that too many people get involved with the god thing, or their character defects, or other aspects.All that is really necessary is to stay sober one day at a time, and when problems are to be faced, to do the next right thing.

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There is no doubt that these excuses for making light of AA treatment comes from the fact that all of AA's suggestions conflict with one's natural progression. So let us describe" how it works"!

We use Mas Q where he starts and Mas A which is his ideal distribution. We will be using two algorithms, algorithms 2. and 3., which will be described here and shown in detail in appendices (b) and (c)., to obtain a Mas for any time period. That Mas is called his natural progression since it does not use any of the suggestions that AA offers other than being welcomed at meetings.

We will investigate the natural Mas of the normal person at the 90 day mark and compare it to what Mas he would have if he seriously started AA treatment from the beginning.

Algorithm 2 moves each mac toward the corresponding ID mac the least amount so that one of the dimensions moves to where it should be at the 90 day mark .We call such a dimension fixed. We use the least amount because it is consistent with everyone's resistance to change. The reason that we need to do this before going to algorithm 3 is that this algorithm leaves four macs untouched, Ex,SI,Se,C. This would be an insurmountable problem if the ideal Mas has different values at these four places. They take on the role of personality traits. If we knew the personality of the person and assumed it did not change over time, then we could just use algorithm 3 to get to our desired Mas. In fact we can see how the Mas changes by giving the person different personality traits.

Algorithm 3 is applied by interchanging two particular macs which fixes the coordinate of a dimension. This interchange gives us a better understanding of these compounds. If the physical dimension P is not fixed by algorithm 2, we interchange W and Ax until the P coordinate is fixed. What we are saying here is that the more time we spend working the less anxiety we will have up to a point and vice versa. Once P is fixed, we interchange Ar and Ax. The more we can reduce Ax, the more we will be able to think more clearly. Now P and E are fixed since P is 2% of Ar and 2% of Ax so the interchange does not effect P. If this fixes I then we are finished since whenever three dimensions are fixed so is the fourth one since the sum is always 1. However this almost never happens. In order to fix I, we interchange Ar and L. Because of the high percent of spirituality,.4, and how there is an interchange with clear thinking, what L represents is the spiritual part of our reasoning. Our myths and fantasies which Joseph Campbell revealed as so necessary for our psyche come into play.

Spirituality increases as practicality and rationality decreases. So long as we are involved with the practicalities of life there is very little imagination. But in order to produce a work of art that is truly inspiring, it must contain both Ar and L and it is the interplay between them the fixes I.

Once I is fixed, both P and E will move from their fixed positions, but not by much, They can be brought back by interchanging M and Re. Since M and Re have the same percent for I, any such interchange will leave I fixed. We see that we can add daily prayers to our mornings if there is time. If not then we make sure we get them in later. In any case, we have P and I fixed. In most cases E will be within one minute of its fixed position and we will have arrived at our Mas.

AA Treatment

In [1], we chose a Mas for N and one for G at the 90 day mark that reflected the fact that both alcoholics were involved with AA treatment from the beginning. Since the behavior of these people differ in a rather extreme way, we set up an empirical rule taken from their dry drunk states and their ideal states that not only matched the 90 day state of each of those people, but derived a 90 day state for any person in Pop(u) to represent that person involved with AA treatment from the beginning. The rule will be given in appendix (d).

Since each application of the matrix takes 18 days, N⁵Q represents the natural Mas at 90 days and N⁵Q(AA) the Mas obtained from being treated by AA from the start. The result is as follows:

	M	W.	Ex	Sl	L	Re	Se	Ax	Ar	C
Q	1	6.25	.5	6.	1.65.	.3.	.4.	7.1.	.3.	.5
N ⁵ Q	1	6.271.	.5047	6.0458	1.7085	.3084	.4009	6.8991	.3398	.5539
N ⁵ Q(AA)	1	6.3417	.45	6.05.	1.4186.	.54	.3.	7.0616	.6381	.2

Looking at each Mas, we see that the treatment requires more W, quite a bit less L, more Re, much more Ar, and much less C, matching AA's list of suggestions. But the most striking difference is in Ax. In the normal progression, Ax is reduced from Q by 14m where with treatment, it is reduced by about 2.5m. Keeping in mind that each Mas produces equivalent states, we have to ask ourselves, "Just what is AA requesting of the dry drunk who first joins?" We assume he just got out of detox. His negative emotions are enormously time consuming. He comes to AA in a desperate state, fighting against his own aversion to being called a "drunk". When he gets there, he meets people who welcome him, empathize with him, honestly want to help him in any way they can. He begins friendships with some of them. His Ax drops day by day; he is feeling better than he has in years. Even his obsession to drink has lessened. He is on a "pink cloud", so they say. Instead of getting a respite he so badly needs, he is told to get a sponsor right away. His sponsor tells him to start working the steps, etc. AA's primary concern is not lowering the person's Ax but rather building a defense at the start against an almost overwhelming obsession to drink again over some small provocation that will happen all too often. He is already well aware of the discrepancy in his thinking between his going out and his remaining sober. He just cannot help it. Going out again

Is completely understandable. It certainly does not represent failure. So long as a person goes back to detox and then to meetings, he is circling around and that cycle of going in and out will continue until he reaches that state of desperation where he will do anything he is told to do to keep himself from going out again. This behavior is an essential part of AA treatment. First time winners are very rare.

Not only is it true that an average alcoholic has less Ax at 90 days if he progressed normally rather than be treated at the start, it is true for every alcoholic in Pop(u). The natural progression produces a smaller Ax at 90 days than does the AA treatment, which can be anywhere from 5m less to almost 1h less. It is almost impossible for anyone to go along with the treatment when his anxiety is being relieved more by just being there and knowing that he is a member of AA.

For the intellectual alcoholic we have the following:

	M	W	Ex	SI	L	Re	Se	Ax	Ar	C
Q	1	6.25	.5	6	1.65	.3	.4	7.1	.3	.5
G^5Q	1	6.1941	.4981	6	1.7348	.2978	.4009	6.9063	.4679	.5
G^5Q(AA)	1	6.25	.5	6	1.65	.38	.3	7.03	.68	.2

Here Ax is lowered by 11.6m compared to 4.2m for AA treatment.

The Quartiles

To get the quartiles of any progression, we again use our starting vector Q and our ID eigenvector from matrix A in $Pop(u)$ and let $R=[A]-[Q]$ which is called the directed range. We let $Q1=1/4R+[Q]$, $Q2=1/2R+[Q]$, $Q3=3/4R+[Q]$. Since it is the emotional marker that we are concentrating on, we find how many applications are necessary in order to obtain the level of emotional time that is within 1m of the third coordinate of $Q1, Q2, Q3$.

From [1], we get the following:

Q1 Q2 Q3

N. 29. 70. 140 : 1.45y. 3.5y. 7y

G. 42. 106 225 : 2.1y 5.3y 11.35y

These values represent significant changes in our emotional serenity. We will first examine how our natural Mas and our AA Mas compare at these milestones. In order to obtain our AA Mas at these values, we will assume that by day 90, AA has been so fixed in the person's behavior, that from then on, he can use his natural progression and the AA changes will stick. This will be verified by our results. To do this, we start at $A^5Q(AA)$ and progress to $[Q_i]$. Tables 2 and 3 give us the results for N and G.

	Q	N ²⁹ Q	N ²⁹ Q(AA)	N ⁷⁰	N ⁷⁰ Q(AA)	N ¹⁴⁰ Q	N ¹⁴⁰ (AA)
M	1	1	1	.976	1	1	1
W	6.25	6,3758	6.4205	6.4998.	6.5325	6.6081.	6.6198
Ex	.5	.5234	.4802	.5472	.5176	.5721	.5543
SI	6	6.2336	6.2413	6.4721	6.478	6.7213	6.7104
L	1.65	2.0105	1.7681	2.3524	2.173	2.6513	2.5482
Re	.3	.3420	.5279	.4090	.5130	.4298	.4983
Se	.4	.4047	.3242	.4094	.3541	.4144	.3343
Ax	7.1	5,8422	6.0132	4.5901	4.7104	3.3536	3.4766
Ar	.3	,4992	.7326	.7011	.8683	.9201	1.006
C	.5	.7686	.4920	1,0430	.8532	1.3295	1.208.

Table 2

	Q	G ⁴² Q	G ⁴² Q(AA)	G ¹⁰⁶ Q	G ¹⁰⁶ Q(AA)	G ²²⁵ Q	G ²²⁵ Q(AA)
M	1	.911	.9284	.8799	.9041	.9200	.936
W	6.25	6.2238	6.2274	6.0588	6.0619	5.665	5.667
Ex	.5	.4979	.4982	.4845	.4847	.4524	.4526
SI	6	6	6	6	6	6	6
L	1.65	2.0452	1.9656	2.1379	2.0768	1.8691	1.8262
Re	.3	.3884	.4581	.4013	.4539	.3229	.3545
Se	.4	.4040	.3028	.4078	.32290	.4238	.3711
Ax.	7.1	5.5772	5.7348	4.0669	4.1994	2.5312	2.6162
Ar	.3	1.4575	1.6792	3.0629	3.2504	5.1356	5.4352
C	.5	.5	.2055	.5	.2459	.5	.3422

Table 3

Looking at these tables, we see that if we take the 90 day Mas representing AA treatment from the beginning, it continues to hold if we take that Mas as the base and take our natural progression from that point. If we compare the two comparable Mas's, we see that W is more, L is less, Re is more, Ar is more, C is less and Ax is more, sure signs that AA treatment is still continuing.

What this means is that the treatment is fixed in our psyche. From that point on, we can continue at our natural pace and AA treatment will remain with us. The additional Ax time could come about by his keeping in mind that he is an alcoholic and that he can go out at any time if he is not careful. The additional Ar time might come about by his continuing to go to meetings. These numbers show that AA's suggestions are still being followed even as he settles into his normal life.

What is also significant is that when we look at Q3 time in both matrices, the normal progression and the AA progression are close to one another and as time goes by they will get closer and closer. It is easy to imagine that eventually the natural progression will take over and the person will stop going to meetings. He will be under the false impression that he is cured from alcoholism and he stops worrying about AA. But one is never cured and without continued treatment it would be the easiest thing in the world to take a drink at what would seem to be an appropriate time. This could have devastating consequences. It is fortunate that so many people with long periods of sobriety feel the need to come back to a meeting. It is interesting to see that the mathematical results of the two progressions converging testifies to the observed fact that people with long term sobriety go out, some over the smallest circumstances.

If we examine the steps carefully, we see that they are divided into three distinct levels of emotional serenity, each level requiring more serenity than the previous one, and each level ending with an action step that can be most readily identified with each quartile.

The first level involves the first four steps. They are all about ourselves. We admit that we are powerless over alcohol. We get to choose our higher power. We let this higher power take away some of the negative emotions about things we have no control over which are so troubling. The action step is step 4, when we make a searching inventory of ourselves, and learn honestly who we are, our defects and our talents. Only then will we be able to change the things we can change.

The second level involves other people. Step 5 requires that we tell

another person about our defects besides our higher power. We are now bringing our social world in to examine our defects and we are going to get feedback which requires us to be more serious than just examining ourselves on our own. We will find defects in our character that are holding us back from having relationships with other people that are necessary for our well being. So we humbly ask our higher power to remove them so that these relationships can take hold. The action step is step 9 where we must make amends to people that we harmed and it is only after we have reached the emotional level of Q2 where such amends will find its true purpose, releasing us from the guilt humiliation, etc., which is necessary to achieve before that level of serenity is acquired.

For all of these 9 steps, the alcoholic always had time to prepare. He could decide when he was ready to make his searching inventory or when he would make an amend to this or that person. But once we work step 10, there is no time for preparation. You say or do something to someone that hurts them, or they do or say something that hurts you, and it has to be taken care of promptly. Otherwise it will fester and cause the alcoholic's anxiety to increase. This level, identified by Q3 requires the most emotional serenity. It's action step is step 12, where we share our strength, faith, and hope with other sick and suffering alcoholics. It is a level that takes a lifetime and is never fully achieved, but it justifies why some alcoholics who have over 40 years sobriety still come to meetings, some every day, to allow this serenity level to continue to move toward its unattainable goal.

We can thus identify the time necessary to achieve each quartile with the earliest times when we should work steps 4,9,12 most effectively.

Appendix (a)

Algorithm 1

From a table of 1 digit percents, we find the matrix that most closely approximates the distribution $[p,i,e]$, $A1(u11,u12,u13)$ whose distribution is $[p1,i1,e1]$. We find $p1-p$ and add it to $u11$ obtaining $u21=p1-p+u11$. Similarly, $u22=i1-i+u12$, $u23=e1-e+u13$ obtaining matrix $A2(u21,u22,u23)$. Continuing in this way, we finally get $A_n(un1,un2,un3)$, where $[pn,in,en,sn]$ are all less than 1'm from the given distribution. We must include s here since it is possible to have p,i,e all come under one minute but s be over one minute, and this is unacceptable.

If we cannot obtain such a matrix, the algorithm will show just where the distribution must be modified.

Appendix (b)

Algorithm 2

Let $[Q]=[p_1, i_1, e_1]$ and $[A]=[p_f, i_f, e_f]$ and $[A^n Q]=[p, i, e]$. We look at $(p-p_1)/(p_f-p_1)$, $(i-i_1)/(i_f-i_1)$, $(e-e_1)/(e_f-e_1)$, $(s-s_1)/(s_f-s_1)$. At least one of these values will be a number between 0 and 1. The smallest such number will be denoted by q because it represents the least amount of change from Q . We line up the corresponding Mas's for Q and A and find $qM(A) + (1-q)M(Q)$, $qW(A) + (1-q)W$, $qEx(A) + (1-q)Ex(Q)$, etc. If q came from coordinate P , then when we evaluate the new Mas we find that P has the value p of $[A^n Q]$. In that case we say P is fixed. If q came from coordinate I , then I would be fixed, etc. When q is applied, we form a modified dry drunk state mQ , and algorithm 3 starts from mQ .

Appendix (c)

Algorithm 3

We assume a modified $mQ[p1,i1,e1]$ and $A^nQ[p,i,e]$. In order to fix P, we first look at the percents for P of W and Ax. For W it is .6 and for Ax it is .02. Let $r = +/-(.6-.02)/24$. We use the plus sign if we are adding time to W, the - sign if we are taking time away from W. Then $(p-p1)/r = t1$ gives us the time exchange that must take place to fix P. Once we have this value, we get new values for I and E: $i1 +/-(.2-.25)/24)t1 = i2$, $e1 +/-(.15-.63)/24)t1 = e2$.

The next step is to vary Ax and Ar. Since both macs have the same P percent, .02, P remains fixed. We seek to fix E. Therefore let $r = +/-(.18-.63)/24$ and find $t2 = (e-e2)/r$ which fixes E. Then P and E are fixed. However $i3 = i2 +/-(.7-.25)/24)t2$ and we have $[p,i3,e]$. In order to fix I, we must interchange time between Ar and L. In this case $r = +/-(.315-.7)/24$ and $t3 = (I-i3)/r$ fixing I. However in this case P and E move away from their fixed positions. For P we have $p' = p +/-(.085-.02)/24)t3$ and $e' = e +/-(.2-.18)/24)t3$ showing us they are not far from their fixed positions. Now we interchange M and Re WE have $r = +/-(.4-.1)/24$ and $t4 = (p-p')/r$. Thus P and I are fixed since I is left untouched by the interchange. In most cases $E' +/-(.2-.15)/24)t4$ will be within one minute of its fixed value.

Appendix (d)

The Ninety Day Rule For AA Treatment

We let $\text{mac}(A^5Q(AA)) = \text{mac}'$, $\text{mac}Q = \text{mac}$, $d(\text{mac}) = \text{mac}(A) - \text{mac}(Q)$

$f(\text{mac}) = \min(d(\text{mac})/2, .1)$, if $d > 0$, or 0, if $d \leq 0$

$g(L) = \min(d(L)/2, .3)$ if $d > 0$, or 0, if $d \leq 0$

$$M' = M - f(M)$$

$$W' = W + f(W)$$

$$Wx' = Ex - f(Ex)$$

$$Sl' = Sl + .05 \text{ if } d > 0, \text{ or } Sl \text{ if } d \leq 0$$

$$L' = L - g$$

$$Re' = (Re(A) + Re)/2 + .15$$

$$Se' = Se - .1$$

$$Ar' = Ar + .38$$

$$C' = C - .3$$

The rest of the time to make up the 24h is put into Ax. If Mas' is not equivalent to A^5Q , then we use the natural progression with Mas' as base to arrive at our final result.